Formal Performance Analysis of Optimal Relays-Based Protection Scheme for Automated Distribution Networks

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Abstract

The dominance of dual-setting directional overcurrent relays (DS-DOCRs) based protection schemes and associated high-reliability requirements require a rigorous verification of these schemes before deployment. Traditionally, numerical and simulation-based methods are used to analyze the performance of these schemes. However, they are incomplete and, thus, cannot provide complete and accurate analysis results due to their inherent limitations, like round-off errors and sampling-based deductions. Analyzing the impact of each replacement level (replacement of conventional DOCRs by DS-DOCRs) on the protection performance/efficacy of individual network lines is challenging as each deployment level leads to different primary and backup relay protection pairs, which may lead to miscoordination. Also, the random and uncertain nature of faults leads to numerous possible scenarios, which need to be rigorously considered during the analysis. As a more complete and accurate analysis approach, we propose to utilize probabilistic model checking which is a formal verification technique, for the performance verification of DS-DOCRs-based protection schemes. This paper presents a case study on the formal verification of a state-of-the-art DS-DOCRs-based protection scheme for power distribution networks using the probabilistic model checker PRISM. The proposed methodology allowed us to determine the most optimal protection solution for different scenarios.

Keywords: Protection Systems, Conventional DOCRs, Dual Setting DOCRs, Probabilistic Model Checking, Formal Verification,

1. Introduction

Increased deployment of distributed generation (DGs) into the power system poses numerous protection-related challenges, such as bidirectional power flow issues, changing/increased short circuit levels, blinding of protection, sympathetic tripping, etc. Directional overcurrent relays (DOCRs) are usually deployed in the DGs integrated meshed distribution networks to address the sympathetic tripping issue caused by the bidirectional power flow [1, 2, 3, 4, 5]. Furthermore, the increased fault current levels can cause severe damage to the equipment and personnel. Similarly, the fault-clearing time of DOCRs is directly related to the number of relays installed, as the time delay settings must incorporate selectivity constraints. Thus, a quicker, more selective, and more reliable protection system is needed to prevent these challenges [4]. With recent technological developments, multi-functional digital relays [6, 7] have been proposed as an alternative to conventional protective relaying. These digital relays incorporated with communication and information infrastructure have paved the way for faster protection schemes [8].

Recently, dual-setting DOCRs (DS-DOCRs) have been proposed to further enhance the protection capabilities of conventional DOCRs (C-DOCRs) and to reduce the total fault-clearing time [9, 10]. Compared to the C-DOCR, a single DS-DOCR can operate simultaneously in two directions, namely primary or forward and backup or reverse order, thus concurrently providing both primary and backup protection. This feature offers an augmented performance by reducing the total operation time of relays. Different algorithms have been proposed by utilizing the DS-DOCRs to protect the microgrids, radial, and meshed distribution networks [9, 10, 11, 12]. The protection system's reliability directly influences the power system's reliability. The extensive usage of C-DOCRs and DS-DOCRs-based protection schemes, along with the associated reliability requirements, highlights the need for a rigorous statistical analysis of the reliability and performance aspects before deployment. This analysis will pave the way to compare different protection schemes and help to optimize the protection algorithms, thus, contributing towards more dependable and secure power systems. Traditionally, protection systems are analyzed using numerical and simulation-based techniques [13, 14]. The numerical methods include computer arithmetic, rounding-off errors, and avoidance of real-time possible/unforeseen situations and thus lead to inaccurate and incomplete analysis results. Likewise, the simulation-based analysis cannot be rigorous due to the high computational cost associated with the thorough analysis. Therefore, a sampling-based approach is used to assess the performance that compromises the completeness of the analysis. This way, some corner cases may be left unnoticed, which can pose devastating consequences to the protection system owing to their safety-critical nature.

On the other hand, formal methods [15], like model checking [15], have been advocated to provide an exhaustive and complete analysis and are thus widely used to overcome the limitations associated with conventional analysis techniques. Model checking is a model-based approach where the system model and its desired properties are fed to the model checker (an automatic verification tool), which thoroughly tracks the entire state-space of the system to evaluate its correctness and desired performance parameters. The verification results can be accurate if the system behavior and specifications are captured correctly. Probabilistic model checking [16] is a variant of model checking that allows verifying probabilistic temporal properties for markovian models. It has been employed for quantitative verification of transmission networks by utilizing the data measured by phasor measurement units (PMUs) as a backup protection system [17]. Similarly, the fault location, isolation, and restoration model integrated with the power line carrier and communication network have been analyzed using the PRISM model checker [18, 19]. Moreover, probabilistic reliability analysis of relay-protected components has also been presented in [20, 21, 22].

Additionally, the discrete-time Markov models of C-DOCRs and DS-DOCRs have been developed in [20]. Rigorous performance analysis of C-DOCRs, DS-DOCRs, and mixed C-DOCRS and DS-DOCRs based protection systems, and their comparison is essential owing to the widespread deployment of these relays in modern protection systems to ensure the uninterrupted supply of power. Therefore, we propose to use the probabilistic model checker PRISM to analyze and compare the performance of C-DOCRs, DS-DOCRs, and mixed C-DOCRs and DS-DOCRs-based protection systems to determine the most optimal protection solution in terms of perfomance and associated cost. We also compare our optimal solution with the simulation-based solution [11]. To the best of our knowledge, no probabilistic verification-based performance analysis method has been proposed to verify the C-DOCR, DS-DOCRs, and mixed C-DOCRs and DS-DOCRs-based protection systems.

1.1. Our Novel Contributions

Our primary contribution in this paper is to present a a methodology for the formal analysis of C-DOCRs, DS-DOCRs, and the hybrid deployment of C-DOCRs and DS-DOCRs based protection systems [11] to demonstrate the usefulness of a methodology in the context of verifying protection systems. We propose to use the probabilistic model checker PRISM for this purpose. Using the formal models of C-DOCRs and DS-DOCRs, we construct the markovian model of the protection system and formally verify the quantitative properties using PRISM. Finally, we provide a performance comparison for different replacement levels of C-DOCRs by DS-DOCRs to determine the most optimum replacement level of C-DOCRs with DS-DOCRs while maintaining a compromise between speed and cost. Our analysis shows the effectiveness of probabilistic model checking for analyzing the protection systems, as it provides valuable insights about the impact of the replacement of C-DOCRs by DS-DOCRs on the isolation success of each line in the network.

1.2. Organization of this Paper

The rest of the paper is structured as follows. Section 2 presents a brief overview of probabilistic model checking, PRISM model checker, and C-DOCRs and DS-DOCRs-based protection. Section 3 describes the proposed methodology for developing the formal model of the protection system and its formal performance analysis. Moreover, Section 4 presents a case study and its detailed formal analysis considering different relay deployment scenarios. Finally, Section 5 concludes the paper.

2. Preliminaries

2.1. Model checking

Model-checking is a model-based automatic formal verification technique used for the verification of reactive systems [15]. The system behavior is captured as a finite-state machine, and associated properties are defined in temporal logic. Both the system model and the properties are fed to the model checker, which exhaustively traverses the entire state-space of the model and verifies whether the given properties hold for the given model or not. Thus, ensuring the 100% completeness of the analysis results. If the property fails, it provides a counterexample, i.e., a trace of possible system failure. One of the concerns associated with model checking is the infamous state-space explosion problem, which is caused by the large or even infinite state-space of the system. The limited time and memory resources make it computationally impossible to traverse the whole state-space of the system. This issue is usually addressed by constructing more abstract, and less complex models of the system.

2.2. *Probabilistic Model Checking and PRISM model Checker*

Probabilistic model checking [15] is an extension of conventional model checking for the quantitative and qualitative verification of the systems that exhibit randomness. PRISM [23] is a widely used probabilistic model-checking tool. The system models are specified using a state-based PRISM language, and the property specification languages available in the PRISM are linear temporal logic (LTL), computational tree logic (CTL), and probabilistic computational tree logic (PCTL) [15]. PRISM gives the verification results as true or false for the LTL properties and an estimate of the probability in the case of PCTL or CTL properties. A set of guarded commands is used to develop the markovian model of a given system. A guard is a predicate over all the system variables, and a transition can occur only if the corresponding guard is true according to the specified transition probabilities. The syntax for the PRISM commands is as follows:

```
[action] guard -> <prob_1>: <update_1>
         +.....+ <prob_n>: <update_n>;
```

Protection systems can be expressed in PRISM language as a set of integrated modules running in parallel. A set of probabilistic guarded commands characterizes the behavior of each module, and a finite set of variables are used to denote the state of each module. We propose using the PRISM tool for this analysis because it supports several probabilistic models and allows us to evaluate the actual probabilities and timing-related properties. These statistics play an essential role in the analysis of protection systems. For example, they allow the protection system engineer to evaluate the efficiency of the protection scheme without any concern about overhead modeling time.

2.3. *C-DOCRs Versus DS-DOCRs-Based Protection*

Placement of C-DOCRs in DG-integrated power networks is usually done to handle the bi-directional power flow issue. Each C-DOCR can provide primary or backup protection in the direction of the fault current. For example, suppose the C-DOCR senses a fault current higher than its specified value, and the direction of the fault current is also forward. In that case, the relay operates and issues a trip signal to its associated breaker. On the other hand, if the fault current direction is not forward, then the relay does not operate. The relay closest to the fault point acts as the primary relay for that particular fault, given that the current thresholds and direction conditions are fulfilled. A relay, farther than the primary relay in terms of physical distance, is termed as the backup when the associated primary relay for the fault fails to trip. Since two separate C-DOCRs are needed to provide the primary and backup protection, this leads to long operation times and various protection challenges. To address this issue, DS-DOCRs are deployed. As the name suggests, two different protection settings are possible. In the case of DS-DOCRs, a single DS-DOCR can simultaneously protect in two directions: primary/forward, and backup/reverse, depending on the fault current direction. Whenever a fault is sensed in the forward direction, the forward settings of the relay are activated, and the relay acts as the primary relay. When the fault is detected in the reverse direction, the reverse settings are activated, and the relay trips the relevant breaker to provide backup protection.

Proper coordination between primary and backup relay pairs in various fault scenarios is needed to ensure selective protection operation. The protection coordination is maintained by introducing a suitable time difference between the relevant relay's operation time. Replacing all C-DOCRs with DS-DOCRs provides faster protection by reducing the total operation time but at the expense of a higher cost. This includes the purchase and installation cost of DS-DOCRs, communication infrastructure, and other components. A hybrid deployment of C-DOCRs and DS-DOCRs is used to avert this issue. Employing different combinations of C-DOCRs and DS-DOCRs results in different primary and backup relay pairs. Let's consider the two cases shown in Figs. 1 and 2 to illustrate this concept further. In Fig. 1, all the relays deployed are C-DOCRs; for a fault on line L1 at point F1, relays R1, and R2 act as the primary relays, whereas relays R4, and R5 provide the backup. In the second case depicted in Fig. 2, some of the C-DOCRs, R3, and R6, are replaced by the DS-DOCRs. When a fault occurs at point F1, R1, and R2 act as the primary relays in this case. At the same time, the reverse direction of DS-DOCRs R3, and R6 provides the backup for R1, and R2, respectively. Since C-DOCR R5 is not coordinating with R1, it may also sense the fault in its forward direction and over-interrupt during the backup operation of relay R6. To address this problem, a blocking signal is sent by relay R6 to relay R5, blocking R5 until the operation of R6. This is done using a dedicated communication link between the relays.

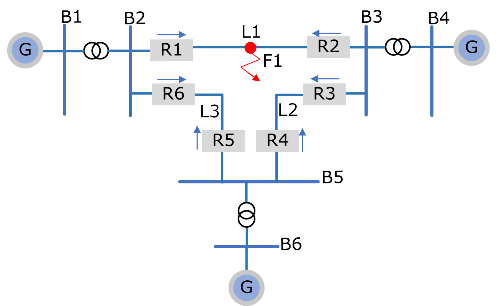


Figure 1: Conventional Relays Deployment Scenario

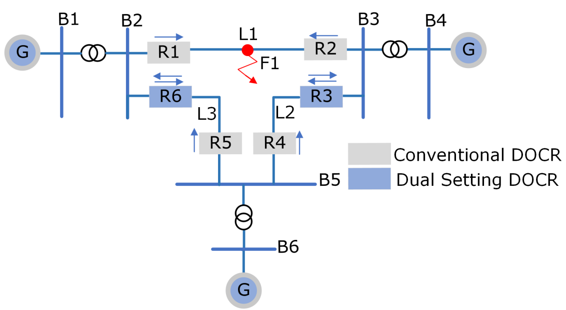


Figure 2: Hybrid Deployment Scenario

3. Proposed Methodology

Performance efficiency is the most desirable feature of a protection system, i.e., selective, fast detection and isolation of faulty sections to ensure an uninterrupted power supply in all fault conditions. Therefore, to determine the most optimal deployment scenario of mixed C-DOCRs and DS-DOCRs, a comparative performance assessment is necessary at each replacement level of C-DOCR by DS-DOCR. Our proposed methodology for modeling and analyzing C-DOCRs and DS-DOCRs-based protection systems is depicted in Fig.3. We have used the discrete-time Markov chains (DTMC) [23] model for modeling purposes, which models time in discrete steps. This choice is motivated by the fact that relays receive their measurements after a discrete time interval. Moreover, the uncertainty and randomness due to faults and the unpredictable nature of relays can be captured using this model type. The proposed methodology is defined step-by-step in the following sections

3.1. Model Construction and Scalability Check

The first phase in our proposed methodology is to build an abstract behavioral model of the given protection system using the PRISM language and analyze it for verification scalability. The system behavior is defined with the help of guarded commands in PRISM. For understanding, let's consider the command below.

```
[] m=1 -> 0.1:(m'=2) + 0.9: (m'=3);
```

The command on the left-hand side of implication $(- >)$ is called the *guard*, and the command on the right-hand side is known as the *update*. Whenever, the *guard* $m=1$ holds, then $(- >)$ with probability 0.1, the next value of m (i.e., m') would be set to 2. With probability 0.9, the next value of m (i.e., m') would be updated to 3.

Given the random and uncertain nature of faults, the associated failure probability of protection components, and the inter-dependency among components, the model's state-space usually becomes very large for complex power networks. Moreover, to determine the optimal protection solution, we have analyzed the performance for all possible deployment levels over a range of possible fault scenarios. If the reachable state-space becomes huge for a given network, then making the analysis becomes computationally impossible. In order to cater for this problem, then the model is simplified and several abstractions are performed, while ensuring that the key characteristics pertaining to the verification problem remain intact, to make the verification for scalable. Ensuring a verification scalability check during the initial stages aids in developing a well-structured model, paving the way for a smooth verification later on.

3.2. Fault Model

The occurrence of faults in power system networks is a random and uncertain phenomenon. However, many different types of faults are possible, which can occur at various locations. In our analysis, we only consider the existence of faults on lines of the power distribution network. A generic representation of the PRISM Fault model is shown in Listing 1. The variable *flt* represents the probability of fault on the system. Once the fault

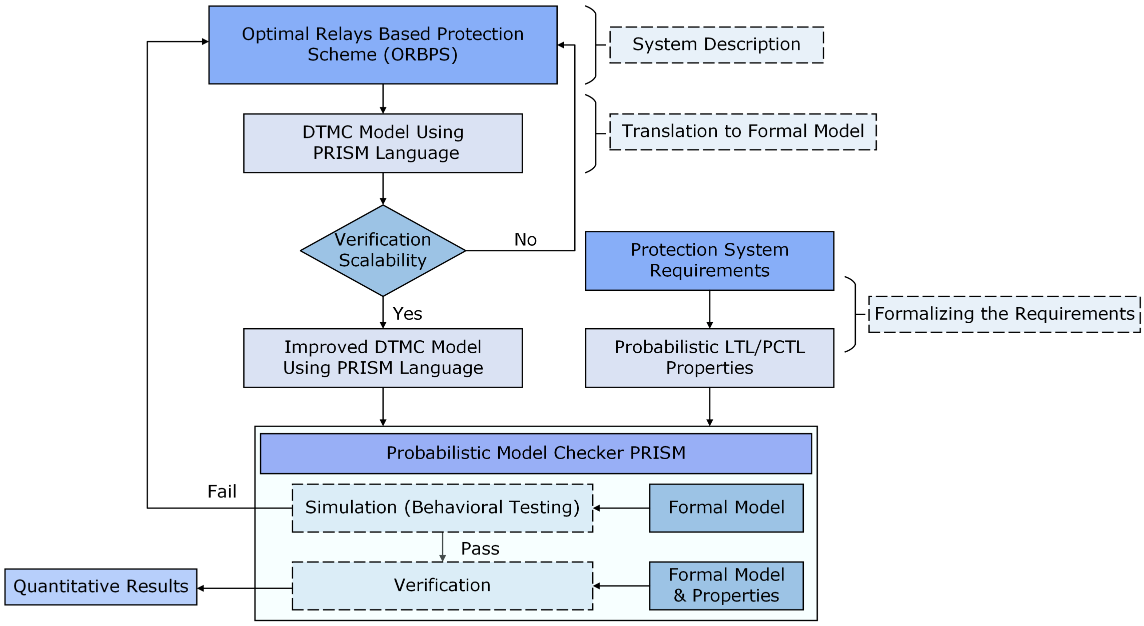


Figure 3: Proposed Methodology

variable *Fault* becomes true, the transition probabilities for its location can be derived based on the total number of lines, "Lines", present in the power network. For example, if there are *n* number of lines in a network that are equally probable for fault location, then the transition probability of the presence of fault on a particular line can be given as *1/n*. Moreover, the Boolean variables *flt1*, *flt2*, etc., represent the existence of fault on the associated lines *L1* and *L2*, respectively.

```
module Fault
Fault: bool init false; flt1: bool init false;
flt2: bool init false;  flt3: bool init false;
fltn: bool init false;
[] Fault=false  ->  flt:(Fault'=true)
                   +1-flt:(Fault'=false);
[] Fault=true   ->  1/Lines:(flt1'=true)
                   +1/Lines:(flt2'=true)
                   +1/Lines:(flt3'=true)
                   :
                   +1/Lines:(fltn'=true);
endmodule
```

Listing 1: Fault Module

3.3. *Protection Model*

A relay-based protection model is shown in Listing 2. Two relays are installed on each line to provide the primary protection, whereas each of the primary relays is also supported by a backup relay located on the adjacent line. Thus, each relay can provide primary protection for the fault on the same line and backup protection upon failure of the relay located on the adjacent line. Once the fault model is initiated and it randomly selects a fault on any line of the power system network, the protection system is activated to isolate the faulty sections from the healthy network. Each relay is called for protection based on the following two conditions:

- *Condition 1*: A fault occurs on the line for which the relay is providing the primary protection.
- *Condition 2*: If the fault occurs on the adjacent line and its relevant primary relay fails to operate, then the backup relay provides the backup protection.

Whenever *Condition 1* or *Condition 2* becomes true, the protection model is activated, and the respective relay is checked for possible internal or external errors. Moreover, if the relay is healthy, it waits for a specific time before operating. After the *t1* or *tb1* time has elapsed, the relay trips (i.e., *r1=4*) and *isol1* become true, indicating that the faulty section has been isolated successfully. On the other hand, if the relay is faulty due to the presence of an internal or external fault, it waits for the *t1* or *tb1* time to elapse, and then goes to failure mode (i.e., *r1=3*). The variable *fail1* becomes true, indicating that the fault is present, but the relay has failed to provide primary or backup protection. Thus, whenever a fault occurs on any line in the network, the protection module invokes the respective primary and backup relay pairs, which operate in a coordinated manner to provide isolation. Protection failure happens only when the primary and backup relay pairs fail to work for a specific fault.

```
module Protection
r1:[0..4]init 0;
int_flt1: bool init false;
ext_flt1: bool init false;
isol1: bool init false;
fail1: bool init false;
c1:[0..10] init 0;
[]r1=0& Condition1=true|Condition2=true  ->
        int_flt:(int_flt1'=true)&(r1'=1)
        +ext_flt:(ext_flt1'=true)&(r1'=1)
        +1-int_flt-d:(r1'=2);
[]r1=1&Condition1=true&c1<t1  ->
        (c1'=c1+1)&(r1'=1);
[]r1=1&Condition1=true&c1=t1  ->
        (fail1'=true)(r1'=3)&(c1'=0)
        &(ext_flt1'=false)&(int_flt1'=false);
[]r1=1&Condition2=true&c1<tb1  ->
        (c1'=c1+1)&(r1'= 1);
[]r1=1&Condition2=true&c1=tb1  ->
        (fail1'=true)&(r1'=3)&(c1'=0)
        &(int_flt1'=false)&(ext_flt1'=false);
[] r1=2&c1<t1&Condition1=true  ->
        (c1'=c1+1)&(r1'=2);
[] r1=2&c1=t1&Condition1=true  ->
        (isol1'=true)&(r1'=4)&(c1'=0);
[] r1=2&c1<tb1&Condition2=true  ->
        (c1'=c1+1)&(r1'=2);
[] r1=2&c1=tb1&Condition2=true  ->
        (isol1'=true)&(r1'=4)&(c1'=0);
endmodule
```

Listing 2: Protection Module

3.4. Simulation

Once the complete formal model of the relay-based protection system is developed, the inbuilt PRISM simulator tab is used to check its functional correctness. The simulator tab allows model debugging by selecting random paths to detect functional errors and undesired scenarios. The model is then updated and finalized by fixing the detected issues before the rigorous, and thus time consuming, formal verification step.

3.5. Performance Properties Verification

The desired performance requirements/properties are specified in PRISM using the PCTL. The updated and finalized formal model is verified against probabilistic properties using the PRISM model checker. Both the formal model and PCTL properties are fed to the PRISM, which exhaustively traverses the entire state space of the model and returns the quantitative verification results in the context of verifying and comparing the C-DOCRs and DS-DOCRs based protection system to determine the most optimal protection solution. Evaluation and comparison of the following performance-related quantitative properties is desired in this context.

```
1. P=? [F<=t Condition1=true & "primary relay
       operated"]
2. P=? [F<=t Condition2=true & "Backup relay
       operated"]
3. P=? [F<=t Condition1=true & "Line is
       isolated"]
```

For each replacement level of C-DOCRs by DS-DOCRs, the properties mentioned above help to evaluate the chances of operation of primary relays, the likelihood of failure of primary relays followed by the operation of backup relays, the probability that any combination of primary and backup relay pair can operate to provide line isolation within first “t” time units, respectively.

4. Case Study: C-DOCRs and DS-DOCRs Based Protection Scheme

To illustrate the usefulness of our proposed methodology, we formally evaluate the performance of the C-DOCRs and DS-DOCRs-based protection scheme proposed by Amin et al. [11], i.e., a typical 9-bus DG-integrated meshed distribution network. A single-line diagram of this test system is shown in Fig. 4. This network comprises of 6 lines L1 to L6, and each line is equipped with two DOCRs. A total of 12 C-DOCRs (R1 to R12) are present in the system, which can be replaced by DS-DOCRs. Further details about the considered system are available in [24]. Using the formal models of C-DOCRs and DS- DOCRs, we have analyzed the performance of this 9-bus protection network. In this analysis, we first deployed C-DOCRs and verified the test model. During the next stage, we analyzed the system's performance by replacing C-DOCRs one at a time with DS-DOCRs. Different penetration levels of DS-DOCRs are considered like 0%, 8.33%, 16.67%, 25%, 33.33%, 47.67%, 50%, 67%, 75%, 83.33%, 97.67%, and 100%. Based on the verification results, the best possible solution is determined.

4.1. Protection System Model

The major step in the verification methodology is to express the behavior of the protection system as a DTMC model. We developed the generic protection model, as shown in Listing 2, which can be used to depict the behavior of any arbitrary protection network. This subsection explains the interaction between different relays integrated to develop a formal model of the complete protection system. A logical block diagram representation of the protection system network of Fig.4

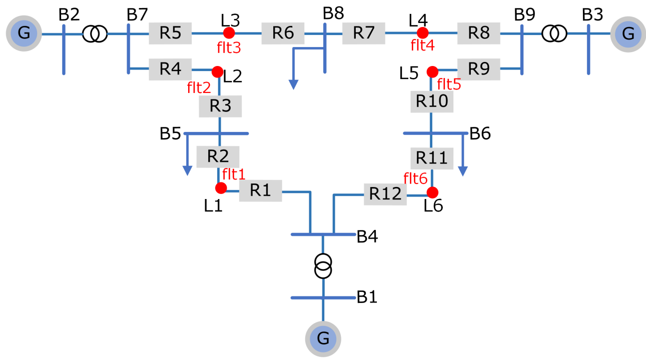


Figure 4: Single Line Diagram of 9-Bus Test System

using all C-DOCRs is given in Fig. 5. A fault module is constructed separately. The power system network comprises six lines (L1-L6), and all the lines are equiprobable for the occurrence of a fault event. Thus, the fault module randomly chooses a fault event on one line among all the lines with a transition probability equal to one divided by the total number of lines (i.e., 1,/6). Two relays are deployed on each line to provide primary protection. The upper block of Fig. 5 represents the interaction between R1, R3, R5, R7, R9, and R11, and the lower block represents the interaction between the relays R2, R4, R6, R8, R10, and R12. So, the corresponding relay protection modules are invoked whenever a fault event is initiated to ensure protection. The protection module of each relay can assess the fault information and the operation status of the adjacent relays following a fault event. Each relay becomes active whenever the associated *Condition1* or *Condition2* becomes true. These conditions are represented by placing the logical AND and logical OR gates at the input side of each relay block in Fig. 5.

To understand the working and coordination between C-DOCRs for the 9-bus network depicted in Fig. 4, let's consider the relay R1. This relay, along with relay R2, provides primary protection for the faults on Line L1 and supports relay R3 as a backup for a fault on Line L2. Moreover, the backup to relay R1 is provided by relay R11. Thus, for the relay R1, the *Condition1* means *flt1* is true (i.e., line L1 is faulty), and *Condition2* holds if line L2 is faulty and the relay R3 has failed to operate (i.e., *flt2=true* and *fail3=true*). Moreover, suppose the *Condition1* holds, and the relay R1 fails to operate due to an internal or external error/fault. In that case, this information is accessed and interpreted by the backup relay R11 as *Condition2*. It is also important to mention that we only consider one backup here. Therefore, the relay R11 won't provide a backup to relay R3. Following a similar pattern, all other relays present in the network provide protection through mutual interaction in a well-coordinated manner.

Next, consider the logical block diagram representation for the relay pairs (R1, R12) and (R2, R3) of the 9-bus protection network that comprises of all DS-DOCRs as shown in Fig. 6. In this case, each DS-DOCR is represented by two blocks, namely R1 for the forward direction protection and R1_rv for the reverse direction operation. For a fault on line L1, the forward directions of DS-DOCRs R1 and R2 provide the primary protection, and the respective backup relays are R12_rv and R3_rv. In this scenario, relay R1 is activated for *Condition1* and R1_rv takes care of *Condition2*. A complete DTMC model for the 9-bus protection system can be built by extending the above-mentioned block diagram following a similar pattern for the remaining DS-DOCR relay pairs (R4, R5), (R6, R7), (R8, R9), and (R10, R11). Moreover, for a mixed-deployment scenario of C-DOCRs and DS-DOCRs, the model is built by interconnecting C-DOCRs and DS-DOCRs according to the replacement scenarios.

4.2. Results and Discussion

We have used version 4.7 of the PRISM model checker and the Linux OS running on a core i5-7200 CPU at 2.71 GHz with 8.00 GB memory for the analysis. The verification is first done for a 9-bus network, as depicted in Fig.5, with the deployment of all conventional relays. Then each C-DOCR is replaced by a DS-DOCR one at a time till 100% deployment of DS-DOCRs is achieved. For comparison, the performance properties are evaluated at each replacement level.

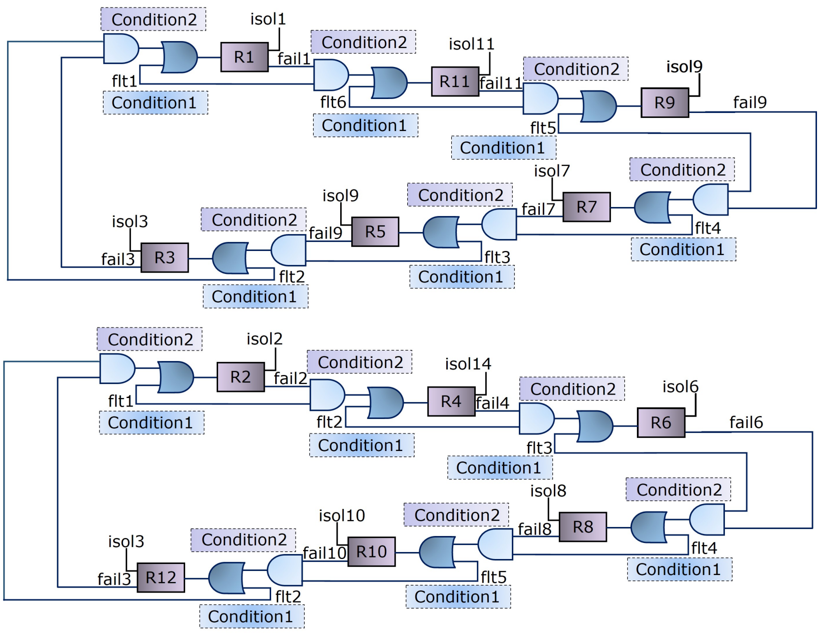


Figure 5: Logical Interconnection Block Diagram for 9-Bus System With C-DOCRs

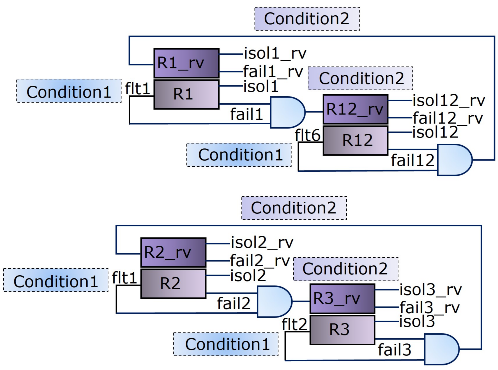


Figure 6: Logical Interconnection Block Diagram With DS-DOCRs

4.2.1. Case I: Performance Analysis of 9-Bus Test System Using the Standard Coordination Approach

In this case, different relay deployment scenarios are assessed for the 9-bus network, assuming a single mid-point fault on each network line. Corresponding to each fault (*flt1-flt6*) location and replacement level of DS-DOCRs, a distinct set of primary and backup relay pairs coordinate to ensure the protection operation as shown in Table 1. The time-bounded probabilities of primary and backup relay operations are determined by utilizing the relay operation times obtained from the standard coordination approach [11]. As the base case, all C-DOCRs are deployed, and the time-bounded primary and backup relay operation probabilities are evaluated for faults on each line as follows:

```
1. P=?[F<=t Condition1=true & (r1=4 &
      isol1=true)]
2. P=?[F<=t Condition2=true & (r11=4 &
      isol11=true)]
```

Moreover, the time-bounded likelihood of a successfulline isolation for each network line is also determined as follows:

```
1. P=?[F<=t "L1_isol"]
2. P=?[F<=t "L2_isol"]
3. P=?[F<=t "L3_isol"]
4. P=?[F<=t "L4_isol"]
5. P=?[F<=t "L5_isol"]
6. P=?[F<=t "L6_isol"]
```

The formal labels *L1_isol*, *L1_isol*, etc. represents the likelihood of successful line isolation following a fault. These labels are further explained in Listing 3. Next, the same process is repeated for each replacement level of C-DOCRs by DS-DOCRs (i.e., 8.3%, 16.6%, 25%, 33.3%, 41.67%, 50%, 58.3%, 66.6%, 75%, 83.3%, 91.6%, and 100%), to assess the impact of the DS-DOCRs on the protection performance of each line. The line-wise isolation success probabilities are shown in Fig. 7. The following points are observed based on the results depicted in Fig. 7a.

Table 1: Primary and Backup Relay Pairs for Different Deployment Levels

DS-DOCRs Deployment Levels		**0%**	**8%**	**16.60%**	**25%**	**33.30%**	**41.60%**	**50%**	**58.30%**	**66.60%**	**75%**	**83.30%**	**91.60%**	**100%**
Fault Locations	**Primary Relays**	**Backup Relays**												
flt1	R1	R11	R11	R11	R11	R11	R11	R11	R11	R11	R11	R12_rv	R12_rv	R12_rv
	R2	R4	R4	R4	R4	R4	R4	R4	R3_rv	R3_rv	R3_rv	R3_rv	R3_rv	R3_rv
flt2	R3	R1	R2_rv	R2_rv	R2_rv	R2_rv	R2_rv	R2_rv	R2_rv	R2_rv	R2_rv	R2_rv	R2_rv	R2_rv
	R4	R6	R6	R6	R6	R6	R6	R5_rv	R5_rv	R5_rv	R5_rv	R5_rv	R5_rv	R5_rv
flt3	R5	R3	R3	R3	R3	R3	R3	R3	R3	R3	R4_rv	R3	R4_rv	R4_rv
	R6	R8	R8	R8	R8	R8	R7_rv	R7_rv	R7_rv	R7_rv	R7_rv	R7_rv	R7_rv	R7_rv
flt4	R7	R5	R5	R5	R5	R5	R5	R5	R5	R5	R5	R5	R6_rv	R6_rv
	R8	R10	R10	R10	R10	R10	R10	R10	R10	R10	R10	R10	R10	R9_rv
flt5	R9	R7	R7	R7	R7	R8_rv	R8_rv	R8_rv	R8_rv	R8_rv	R8_rv	R8_rv	R8_rv	R8_rv
	R10	R12	R12	R11_rv	R11_rv	R11_rv	R11_rv	R11_rv	R11_rv	R11_rv	R11_rv	R11_rv	R11_rv	R11_rv
flt6	R11	R9	R9	R9	R10_rv	R10_rv	R10_rv	R10_rv	R10_rv	R10_rv	R10_rv	R10_rv	R10_rv	R10_rv
	R12	R2	R2	R2	R2	R2	R2	R2	R2	R1_rv	R1_rv	R2	R1_rv	R1_rv

```
label "L1_isol" = (flt1=true &((r1=4|r11=4)
                  &(r2=4|r4=4)));
label "L2_isol" = (flt2=true &((r3=4|r1=4)
                  &(r4=4|r6=4)));
label "L3_isol" = (flt3=true &((r5=4|r3=4)
                  &(r6=4|r8=4)));
label "L4_isol" = (flt4=true &((r7=4|r5=4)
                  &(r8=4|r10=4)));
label "L5_isol" = (flt5=true &((r9=4|r7=4)
                  &(r10=4|r12=4)));
label "L6_isol" = (flt6=true &((r11=4|r9=4)
                  &(r12=4|r2=4)));
```

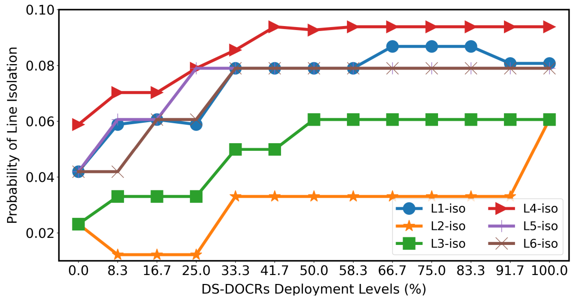
Listing 3: Formal Labels of Properties

- Line L4 has the highest isolation probabilities meaning that it has the lowest protection operation time during all the deployment scenarios.
- Line L2 has the lowest probabilities compared to the other lines, which remains constant beyond 33.3% deployment levels.
- Line L3 has shown a slightly better response than line L2, and no improvement is observed beyond 50% replacement levels.
- Isolation probabilities for the lines L1, L5, and L6 almost saturate beyond 33.3% deployment levels.

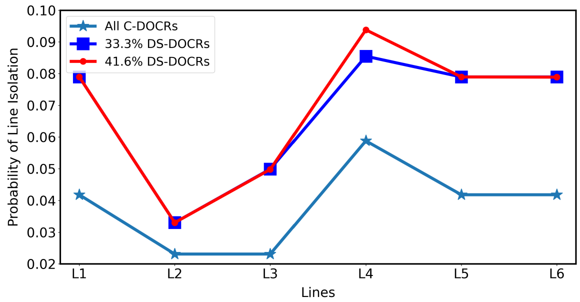
Moreover, it is obvious from Fig. 7b that the protection performance of each line has significantly improved compared to the conventional deployment scenario. Lines L1, L2, L3, L5, and L6 show the same results for 33.3% and 41.6% replacement levels. A substantial improvement is noticed for line L4. Finally, the average probabilities of primary and backup relay operation and lines isolation for different penetration levels are depicted in Fig.8. Overall a rise in relay operation probabilities is noticed up to 25% deployment level, which tends to saturate for the higher penetration levels as depicted in Fig. 8a. Furthermore, the average line isolation probabilities represent a significant improvement up to 33.3% replacement levels, as shown in Fig. 8b. Also, the obtained results are comparable for 33.3% and 41.6%. For 33.3% replacement level R2, R8, R10, and R11 relays are used as DS-DOCRs and the solution vector for 41.6% renders R2, R7, R8, R10, and R11 as DS-DOCRs. Amin et al. [11] reported 41.6% deployment level as the best-compromised solution. On the other hand, our analysis shows that deploying 33.3% replacement level gives the same results for all the lines, and 41.6% DS-DOCRs only improves the line L4 results. Based on this comparison, it is obvious that the deployment of DS-DOCRs increases the relay operation and line isolation probability more significantly at the first stages of the replacement levels. However, the probabilities of relay operation and line isolation saturate as the penetration of DS-DOCRs grows.

4.2.2. *Case II: Performance Analysis of 9-Bus Test System using the Non-Standard Coordination Approach*

In this case, the relay operation times obtained from the non-standard coordination approach are used to evaluate the time-bounded probabilities of primary and backup relay operations [11]. As obvious from Fig. 10,

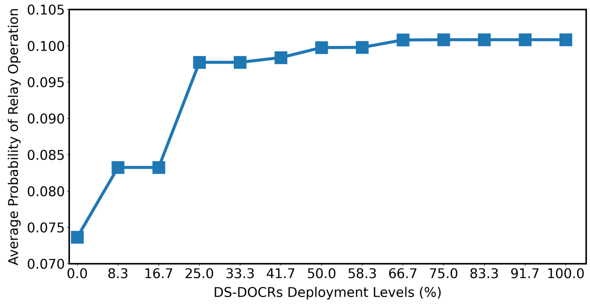


(a)

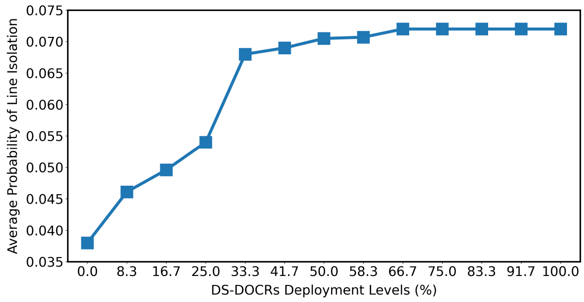


(b)

Figure 7: Impact of DS-DOCRs Deployment Levels on Lines Isolation Success for Case I: (a) Lines Isolation Success Probabilities w.r.t Each Deployment Level and (b) Lines Isolation Success for 0%, 33.3%, 41.6% Deployment Levels

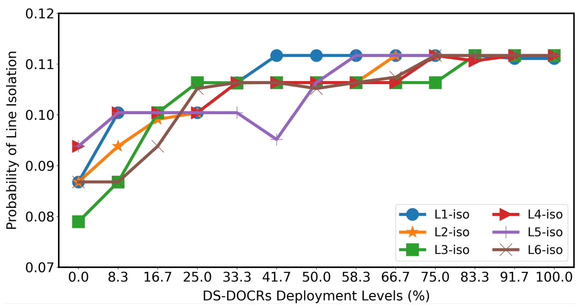


(a)

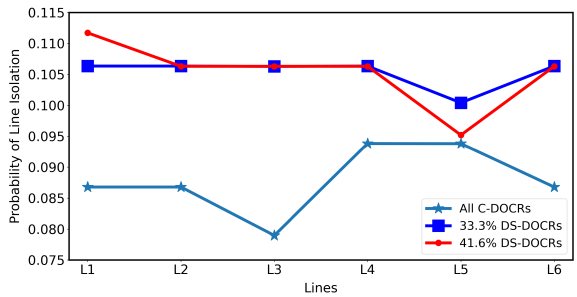


(b)

Figure 8: Optimal Solution for Case I: (a) Average Relay Operation Probability and (b) Average Line Isolation Probability



(a)



(b)

Figure 9: Impact of DS-DOCRs Deployment Levels on Lines Isolation Success for Case II: (a) Lines Isolation Success Probabilities w.r.t Each Deployment Level and (b) Lines Isolation Success for 0%, 33.3%, 41.6% Deployment Levels

an overall increase in the line isolation success probabilities is attained using this approach compared to the standard approach. Fig. 9a shows that the isolation success of lines L2, L3, L4, and L6 remain the same from 33.3% to 58.3% deployment levels. Additionally, the lines L2, L3, L4, L5, and L6 show the same or better performance (for line L5) results for 33.3% deployment level than 41.6% replacement level as shown in Fig. 9b. Finally, the average line isolation probabilities depicted in Fig. 10a indicate that no substantial improvement in line isolation probabilities is attained after the 33.3% replacement level. Thus, the 33.3% deployment level provides the best-compromised protection solution. The solution vectors corresponding to the 33.3% deployment level are R2, R6, R7, and R11. A comparison of the results with those of Case I, given in Fig. 11, indicates that although Case II uses a lesser number of DS-DOCRs, it provides 2.76 times higher line isolation success probabilities compared to the conventional deployment scenario. Moreover, using the standard coor-

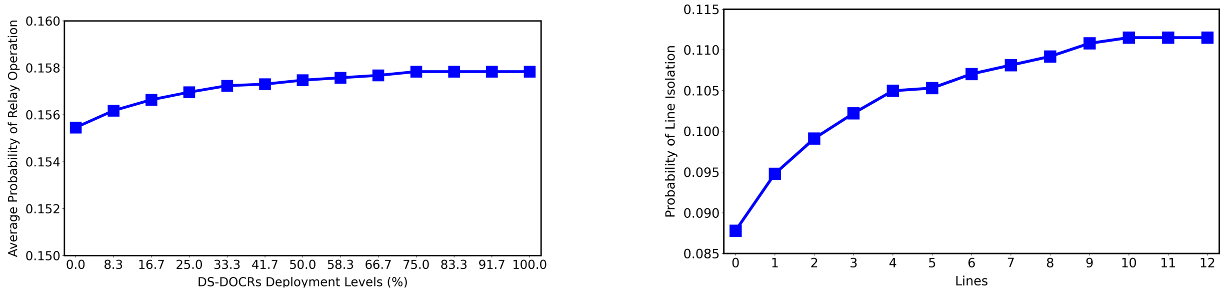


Figure 10: Optimal Solution for Case II: (a) Average Relay Operation Probability and (b) Average Line Isolation Probability

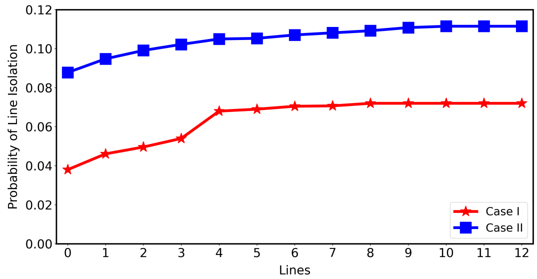


Figure 11: A Comparison of Case I and Case II

dination approach, 33.3%, and 41.6% replacement level yield the same results for all the lines except line L4. On the other hand, using the non-standard approach, lines L2, L3, L4, and L6 show the same results for 33.3% and 41.6% deployment scenarios.

5. CONCLUSION

This paper presents a formal analysis methodology to determine the optimal replacement level of C-DOCRs by DS-DOCRs. A stochastic fault model and protection model capturing the effects of the failure of primary relays and calling for backup relays, is integrated in our model. Moreover, we have identified some performance properties to analyze the impact of different relay deployment scenarios on the protection system performance. The probabilistic model checker PRISM is used to evaluate the protection performance of each distribution network line corresponding to each replacement level. A detailed quantitative insight analyzing the impact of each replacement on the protection performance of individual lines is the distinguishing feature of this approach compared to the traditional technique. Thus, the protection/planning engineer can opt for a specific deployment level according to the protection requirements of individual network lines without compromising the efficiency and cost of the protection solution. Moreover, in the future, we plan to develop and integrate a communication model with C-DOCRs and DS-DOCRs models. This will help to determine the false relay tripping scenarios occurring within an expected time interval due to miscommunication or miscoordination among the first and second-level backup relays while considering single and multiple fault events. Further, this idea can be extended to model, analyze, and scale to modern multi-micro-grid power systems.

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